

# Threshold resummation for exclusive $B$ meson decays

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## Abstract

We argue that double logarithmic corrections  $\alpha_s \ln^2 x$  need to be resummed in perturbative QCD factorization theorem for exclusive  $B$  meson decays, when the end-point region with a momentum fraction  $x \rightarrow 0$  is important. These double logarithms, being of the collinear origin, are absorbed into a quark jet function, which is defined by a matrix element of a quark field attached by a Wilson line. The factorization of the jet function from the decay  $B \rightarrow \gamma l \bar{\nu}$  is proved to all orders. Threshold resummation for the jet function leads to a universal, *i.e.*, process-independent, Sudakov factor, whose qualitative behavior is analyzed and found to smear the end-point singularities in heavy-to-light transition form factors.

## I. INTRODUCTION

Perturbative QCD (PQCD) factorization theorem for exclusive processes [1] states that a hadronic form factor at large momentum transfer is expressed as convolution of a hard amplitude with hadron distribution amplitudes. However, the PQCD evaluation of the pion form factor suffers the soft enhancement from the end point of a momentum fraction  $x \rightarrow 0$  [2]. In this region the hard amplitude is characterized by a low scale, such that perturbative expansion in terms of a large coupling constant  $\alpha_s$  is not self-consistent. More serious end-point singularities, logarithmic and linear, have been observed in the leading-twist (twist-2) and next-to-leading-twist (twist-3) contributions to the  $B \rightarrow \pi$  transition form factors [3–5], respectively.

We argue that when the end-point region is important, the double logarithms  $\alpha_s \ln^2 x$  from radiative corrections should be organized to all orders in order to improve perturbative expansion. These double logarithms have been found in the semileptonic decay  $B \rightarrow \pi l \bar{\nu}$  [4] and in the radiative decay  $B \rightarrow \gamma l \bar{\nu}$  [6], and resummed. Here we shall give a systematic treatment of this type of double logarithms, which is of the collinear origin, by introducing a quark jet function into PQCD factorization theorem. The all-order factorization of the jet function from the decay  $B \rightarrow \gamma l \bar{\nu}$  is proved following the procedures proposed in [7,8], which provides a solid theoretical ground for the modified formalism appropriate for the end-point region. It will be shown that the jet function is defined as a matrix element of a quark field attached by a Wilson line.

The double logarithms in the jet function are resummed into a Sudakov form factor in the Mellin space using the technique developed in [9–12]. In principle, with the definition of the jet function constructed in this work, its threshold resummation can be performed up to the next-to-leading-logarithm accuracy. Since our purpose is to understand the qualitative behavior of the jet function at small  $x$ , it suffices to derive only the leading-logarithm result. Extending the above formalism to the semileptonic decay  $B \rightarrow \pi l \bar{\nu}$ , we obtain the identical Sudakov factor, implying its universality. By means of the inverse Mellin transformation, the Sudakov factor is found to vanish quickly as  $x \rightarrow 0$ . The suppression is so strong that the end-point singularities in the  $B \rightarrow \pi$  form factors are smeared. We conclude that in a self-consistent perturbative evaluation of heavy-to-light transition form factors, the end-point singularities do not exist.

It has been argued that the dependence on the transverse momentum  $k_T$  carried by the light spectator in a  $B$  meson can not be dropped from hard amplitudes for exclusive  $B$  meson decays [6]. As considering higher-order corrections to a hard amplitude,  $k_T$  appears in the ratio  $k_T/k^+$ ,  $k^+$  being the longitudinal component of the light spectator momentum. Obviously, this ratio is not power-like, because  $k_T$  and  $k^+$  are of the same order of magnitude. Hence, a reliable formalism for exclusive  $B$  meson decays must involve the parton transverse degrees of freedom. The effect due to the inclusion of  $k_T$  has been explored thoroughly in [13–15], which also smears the end-point singularities in

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the heavy-to-light form factors [16]. In this paper we shall concentrate only on the Sudakov effect from threshold resummation, and neglect the  $k_T$  dependence.

In Sec. II we work out the  $O(\alpha_s)$  factorization formula for the  $B \rightarrow \gamma l \bar{\nu}$  decay amplitude at the end point. The all-order factorization is proved in Sec. III, leading to the convolution of the hard amplitude with the  $B$  meson distribution amplitudes and with the jet function. Threshold resummation is done in Sec. IV by solving an evolution equation for the jet function. In Sec. V we extract the jet function from the semileptonic decay  $B \rightarrow \pi l \bar{\nu}$ . The smearing effect from the Sudakov factor on the end-point singularity is demonstrated numerically by evaluating the leading-twist contribution to the  $B \rightarrow \pi$  form factor. Section VI is the conclusion. In Appendix A we comment on the Sudakov resummation performed in [4].

## II. $O(\alpha_s)$ FACTORIZATION

We study the radiative decay  $B \rightarrow \gamma l \bar{\nu}$ . The  $B$  meson carries the momentum  $P_1 = (M_B/\sqrt{2})(1, 1, \mathbf{0}_T)$  and the outgoing photon carries the momentum  $P_2 = (M_B/\sqrt{2})(0, \eta, \mathbf{0}_T)$  with the energy fraction  $\eta$ , where the light-cone variables have been adopted. Consider the kinematic region with small  $q^2 = (1 - \eta)M_B^2$ ,  $q = P_1 - P_2$  being the lepton pair momentum, *i.e.*, with large  $\eta$ , where PQCD is applicable. The mass difference between the  $B$  meson and the  $b$  quark,  $\bar{\Lambda} = M_B - m_b$ , is treated as a small scale in the heavy quark limit. As stated in [7], there are two types of infrared divergences in radiative corrections, soft and collinear. Soft divergences come from the region of a loop momentum  $l$ , where all its components vanish. Collinear divergences are associated with a massless parton carrying a momentum of order  $M_B$ . The  $O(\alpha_s)$  factorization of the soft divergences into the  $B$  meson distribution amplitudes has been presented in [7]. Here we derive only the  $O(\alpha_s)$  factorization of the collinear divergences at the end point into the jet function.

According to leading-twist PQCD factorization theorem, the form factor relevant to the decay  $B \rightarrow \gamma l \bar{\nu}$  is written as

$$G(q^2) = \sum_{m=+,-} \phi_m(x) \otimes H_m(x, \eta), \quad (1)$$

where the symbol  $\otimes$  denotes the convolution over the spectator momentum fraction  $x = k^+/P_1^+$ ,  $k$  being the spectator quark momentum. The light-cone  $B$  meson distribution amplitudes  $\phi_m$  are defined by [7,8]

$$\phi_{\pm}(x) = \int \frac{dy^-}{2\pi} e^{ixP_1^+y^-} \langle 0 | \bar{q}(y^-) \gamma_5 (\not{v} + I) \gamma^{\pm} \exp \left[ -ig \int_0^{y^-} dz n_- \cdot A(z n_-) \right] b_v(0) | B(P_1) \rangle, \quad (2)$$

with the dimensionless vectors  $v = P_1/M_B$  and  $n_- = (0, 1, \mathbf{0}_T)$ , and the rescaled  $b$  quark field  $b_v$ . The hard amplitudes  $H_m$  are obtained by contracting the quark-level diagrams with the spin structures corresponding to  $\phi_m$  [17]. Figure 1 is the lowest-order example, where the upper line represents a  $b$  quark and  $\times$  represents a weak decay vertex. The contributions to the hard amplitudes from Figs. 1(a) and 1(b) scale like  $1/(\bar{\Lambda}M_B)$  and  $1/M_B^2$ , respectively. Below we shall concentrate on Fig. 1(a).

The factorization formula in Eq. (1) is appropriate for the region with  $k^+ \sim O(\bar{\Lambda})$ , in which the only infrared divergences are the soft ones absorbed into  $\phi_{\pm}$  in Eq. (2). Near the end point  $k^+ \sim O(\bar{\Lambda}^2/M_B)$ , the internal quark in Fig. 1(a) carries a large momentum  $P_2 - k$  with its invariant mass vanishing like  $(P_2 - k)^2 = -2xP_1 \cdot P_2 \sim O(\bar{\Lambda}^2)$ . This kinematics is similar to the threshold region of deeply inelastic scattering (DIS) with the Bjorken variable  $x_B \rightarrow 1$ , where the scattered quark also carries a momentum with a large component and possesses a small invariant mass  $(1 - x_B)s$ ,  $s$  being the center-of-mass energy. In this region the scattered quark produces a jet of particles, to which the radiative corrections contain additional collinear divergences. Hence, a jet function needs to be introduced into the standard factorization theorem for DIS [18]. Similarly, a jet function has been incorporated into the factorization of direct photon production at a large photon transverse momentum (threshold) [19]. Here we associate a jet function with the internal quark at the end point of the momentum fraction involved in the decay  $B \rightarrow \gamma l \bar{\nu}$ .

An additional collinear divergence from the loop momentum parallel to  $P_2$  appears in the higher-order correction to the weak decay vertex shown in Fig. 1(c). This divergence can be extracted by replacing the  $b$  quark line by an eikonal line in the direction of  $n_+$ :

$$\frac{\not{P}_1 - \not{k} + \not{l} + m_b}{(P_1 - k + l)^2 - m_b^2} \gamma^{\beta} b(P_1 - k) \approx \frac{n_+^{\beta}}{n_+ \cdot l} b(P_1 - k), \quad (3)$$

where  $b(P_1 - k)$  represents the  $b$  quark spinor with the momentum  $P_1 - k$  and  $n_+ = (1, 0, \mathbf{0}_T)$  is a dimensionless vector. The above replacement makes sense, since  $\gamma^\beta$  is dominated by its plus component  $\gamma^+$ ,  $l^-$  is much larger than  $l^+$ , and  $\not{l}\gamma^\beta$  diminishes. The factorization of the fermion flow is achieved by inserting the Fierz identity,

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma_5)_{ik}(\gamma_5)_{lj} + \frac{1}{4}(\gamma_\alpha)_{ik}(\gamma^\alpha)_{lj} + \frac{1}{4}(\gamma_5\gamma_\alpha)_{ik}(\gamma^\alpha\gamma_5)_{lj} + \frac{1}{8}(\sigma_{\alpha\beta})_{ik}(\sigma^{\alpha\beta})_{lj} , \quad (4)$$

in which the first and last terms contribute in the combined structure,

$$I_{ij}I_{lk} \rightarrow \frac{1}{4}I_{ik}(\not{n}_+ \not{n}_-)_{lj} . \quad (5)$$

Assigning the identity matrix  $I$  in Eq. (5) to the trace for the hard amplitude, we obtain Fig. 1(a). The matrix  $\not{n}_+ \not{n}_-/4$  then leads to the loop integral,

$$\begin{aligned} J_{\parallel}^{(1)} &= -ig^2C_F \int \frac{d^4l}{(2\pi)^4} \frac{1}{4} \text{tr} \left[ \not{n}_+ \not{n}_- \gamma_\beta \frac{\not{P}_2 - \not{k} + \not{l}}{(P_2 - k + l)^2} \right] \frac{n_+^\beta}{n_+ \cdot ll^2} , \\ &= -\frac{\alpha_s}{4\pi} C_F \ln^2 x + \dots , \end{aligned} \quad (6)$$

where  $C_F = 4/3$  is a color factor. To obtain the above double logarithm, we have regularized the collinear pole from  $l$  parallel to  $n_+$  by allowing  $n_+$  to possess a small amount of minus component. The correction to the photon vertex in Fig. 1(d) contains only the single logarithm  $\alpha_s \ln x$ , since the phase space of the loop momentum is restricted to  $0 < l^+ < k^+ \sim O(\bar{\Lambda}^2/M_B)$ . The self-energy correction in Fig. 1(e), generating also the single logarithm, is factorized into  $J_{\perp}^{(1)}$  trivially by applying the Fierz transformation in Eq. (5). The explicit expression of  $J_{\perp}^{(1)}$  is not essential here. The reason for labeling the  $O(\alpha_s)$  jet functions by  $\parallel$  and  $\perp$  will become clear in the next section. To group Figs. 1(c) and 1(e), we introduce a quark jet function with the zeroth-order expression and the  $O(\alpha_s)$  expression,

$$J^{(0)} = \text{tr}(\not{n}_+ \not{n}_-)/4 = 1 , \quad J^{(1)} = J_{\parallel}^{(1)} + J_{\perp}^{(1)} , \quad (7)$$

respectively.

It has been shown that the soft divergences in Figs. 1(c), 1(d) and 1(f)-1(h) are absorbed into the  $O(\alpha_s)$   $B$  meson distribution amplitudes  $\phi_{\pm}^{(1)}$  [7]. The remaining infrared finite  $O(\alpha_s)$  contributions, including the single logarithms except that from Fig. 1(e), are assigned into the hard amplitudes  $H_{\pm}^{(1)}$ . Therefore, the modified factorization in the end-point region is written, up to  $O(\alpha_s)$ , as

$$G^{(0)} + G^{(1)} = \sum_{m=+,-} [1 + \phi_m^{(1)}] \otimes [H_m^{(0)} + H_m^{(1)}] \otimes [1 + J^{(1)}] + O(\alpha_s^2) , \quad (8)$$

with  $G^{(0)}$  and  $G^{(1)}$  displayed in Fig. 1. Note that  $J$  does not depend on the subscript  $m$ , a fact obvious from the derivation of Eq. (6).

### III. ALL-ORDER FACTORIZATION

In this section we prove the factorization theorem for the radiative decay  $B \rightarrow \gamma l \bar{\nu}$  at the end point to all orders, and construct the definition of the jet function  $J(x)$  via

$$J(x) \bar{u}(P_2 - k) \equiv \langle u(P_2 - k) | \bar{q}(0) \frac{1}{4} \not{n}_+ \not{n}_- \exp \left[ -ig \int_{-\infty}^0 dz n_+ \cdot A(z n_+) \right] | 0 \rangle . \quad (9)$$

The spinor  $u(P_2 - k)$  is associated with the internal quark, through which the momentum  $P_2 - k$  flows. The idea of the proof is based on induction [7,8] with the help of the Ward identity,

$$l_\mu G^\mu(l, k_1, k_2, \dots, k_p) = 0 , \quad (10)$$

where  $G^\mu$  is a physical amplitude with an external gluon carrying the momentum  $l$  and with  $p$  external quarks carrying the momenta  $k_1, k_2, \dots, k_p$ . All these external particles are on mass shell.

The  $O(\alpha_s)$  factorization of the collinear divergences has been worked out in Sec. II. Assume that the factorization theorem holds up to  $O(\alpha_s^N)$ :

$$G = \sum_{m=+,-} \phi_m \otimes H_m \otimes J, \quad (11)$$

with the definitions,

$$G = \sum_{i=0}^N G^{(i)}, \quad \phi_m = \sum_{i=0}^N \phi_m^{(i)}, \quad H_m = \sum_{i=0}^N H_m^{(i)}, \quad J = \sum_{i=0}^N J^{(i)}, \quad (12)$$

$G^{(i)}$  denotes the full diagrams of  $O(\alpha_s^i)$ . The  $B$  meson distribution amplitudes  $\phi_m^{(i)}$  and the jet function  $J^{(i)}$  are defined by the  $O(\alpha_s^i)$  terms in the perturbative expansions of Eq. (2) and of Eq. (9), respectively. The  $O(\alpha_s)$  hard amplitudes  $H_m^{(i)}$  do not contain the initial-state soft divergences and the end-point double logarithms, which have been collected into the  $B$  meson distribution amplitudes and into the jet function, respectively. We then have the relations,

$$G^{(k)} = \sum_{m=+,-} \sum_{i=0}^k \sum_{j=0}^{k-i} \phi_m^{(i)} \otimes H_m^{(k-i-j)} \otimes J^{(j)}, \quad k = 0, 1, \dots, N. \quad (13)$$

Below we prove the collinear factorization of the  $O(\alpha_s^{N+1})$  diagrams  $G^{(N+1)}$  into the convolution of the  $O(\alpha_s^N)$  diagrams  $G^{(N)}$  with the  $O(\alpha_s)$  jet function  $J^{(1)}$ . Look for the gluon in a complete set of  $O(\alpha_s^{N+1})$  diagrams  $G^{(N+1)}$ , one of whose ends attaches the lower most vertex on the internal quark line. Such a gluon exists, because  $G^{(N+1)}$  are the finite-order diagrams. Let  $\alpha$  denote the lower most vertex, and  $\beta$  denote the attachments of the other end of the identified gluon inside the rest of the diagrams. There are two types of collinear configurations associated with this gluon, depending on whether the vertex  $\beta$  is located on an internal line with a momentum along  $P_2$ . The fermion propagator adjacent to the vertex  $\alpha$  is proportional to  $\not{P}_2$  in the collinear region with the loop momentum  $l$  parallel to  $P_2$ . If  $\beta$  is not located on a collinear line along  $P_2$ , the component  $\gamma^-$  in  $\gamma^\alpha$  and the plus component of the vertex  $\beta$  give the leading contribution. If  $\beta$  is located on a collinear line along  $P_2$ ,  $\beta$  can not be plus, and both  $\alpha$  and  $\beta$  label the transverse components. This configuration is the same as of the self-energy correction to an on-shell particle.

According to the above classification, we decompose the tensor  $g_{\alpha\beta}$  appearing in the propagator of the identified gluon into

$$g_{\alpha\beta} = \frac{n_{+\alpha} l_\beta}{n_+ \cdot l} - \delta_{\alpha\perp} \delta_{\beta\perp} + \left( g_{\alpha\beta} - \frac{n_{+\alpha} l_\beta}{n_+ \cdot l} + \delta_{\alpha\perp} \delta_{\beta\perp} \right). \quad (14)$$

The first term on the right-hand side of Eq. (14) extracts the first type of collinear divergences, since the light-like vector  $n_{+\alpha}$  selects the minus component of  $\gamma^\alpha$ , and  $l_\beta$  in the collinear region selects the plus component of the vertex  $\beta$ . The second term extracts the second type of collinear divergences. The last term does not contribute a collinear divergence. We shall concentrate on the factorization corresponding to the first term, and the factorization corresponding to the second term can be achieved simply by requiring gauge invariance [8].

The identified collinear gluon with  $\alpha = -$  and  $\beta = +$  does not attach the internal quark line directly, which carries a momentum along  $P_2$  at the end point. That is, those diagrams with Fig. 1(e) as the  $O(\alpha_s)$  subdiagram are excluded from the set of  $G^{(N+1)}$  as discussing the first type of collinear configurations. Applying the Fierz transformation in Eq. (5) to break the fermion flow, we have the physical amplitude, in which the two on-shell quarks and the on-shell gluon carry the momenta  $\xi P_2$ ,  $\xi P_2 - l$  and  $l$ , respectively. The fraction  $\xi$  reflects that the momentum flowing through the internal quark is almost parallel to  $P_2$ . Figure 2(a), describing the Ward identity, contains a complete set of contractions of  $l_\beta$  represented by arrows, since the second diagram has been added back. The cuts on the internal quark lines denote the insertion of the Fierz identity.

The second diagram in Fig. 2(a) gives

$$l_\beta \xi \not{P}_2 \gamma^\beta \frac{1}{\xi \not{P}_2 - \not{l}} = \xi \not{P}_2 (\not{l} - \xi \not{P}_2 + \xi \not{P}_2) \frac{1}{\xi \not{P}_2 - \not{l}} = -\xi \not{P}_2, \quad (15)$$

where the factor  $\xi \not{P}_2$  at the beginning of the above expression comes from the internal quark propagator adjacent to the photon vertex, and the term  $-\xi \not{P}_2$  at the end leads to the  $O(\alpha_s^N)$  diagrams. The diagrams  $G_{\parallel}^{(N+1)}$  corresponding to the first term in Eq. (14) are then factorized according to Fig. 2(b). The factor  $n_{+\alpha}/n_+ \cdot l$  is just the Feynman rule associated with the Wilson line along the vector  $n_+$  in Eq. (9). We arrive at the convolution for the first type of collinear configurations,

$$G_{\parallel}^{(N+1)} \approx G^{(N)} \otimes J_{\parallel}^{(1)}, \quad (16)$$

with  $J_{\parallel}^{(1)}$  given in Eq. (6).

The above procedures are applicable to the  $O(\alpha_s^{j+1})$  jet function  $J^{(j+1)}$ . We identify the gluon in a complete set of  $O(\alpha_s^{j+1})$  diagrams  $J^{(j+1)}$ , one of whose ends attaches the lower most vertex  $\alpha$  on the internal quark line. The other end attaches the vertex  $\beta$  inside the rest of the diagrams. For the first term on the right-hand side of Eq. (14), there exists a Ward identity similar to Fig. 2(a). It is then trivial to obtain the factorization of the jet function corresponding to the first type of collinear configurations,

$$J_{\parallel}^{(j+1)} \approx J^{(j)} \otimes J_{\parallel}^{(1)}. \quad (17)$$

The rest part of the proof is exactly the same as of the factorization theorem for the semileptonic decay  $B \rightarrow \pi l \bar{\nu}$  in Sec. IV B of [8], with the pion distribution amplitudes replaced by the jet function, both of which are of the collinear origin. Employing Eqs. (13), (16) and (17) and following the steps of factorizing the decay  $B \rightarrow \pi l \bar{\nu}$ , we derive

$$G^{(N+1)} = \sum_{m=+,-} \sum_{i=0}^{N+1} \sum_{j=0}^{N+1-i} \phi_m^{(i)} \otimes H_m^{(N+1-i-j)} \otimes J^{(j)}, \quad (18)$$

with the  $O(\alpha_s^{N+1})$  hard amplitude  $H_m^{(N+1)}$  being infrared finite. Equation (18) indicates that all the soft and collinear divergences at the end point in the radiative decay  $B \rightarrow \gamma l \bar{\nu}$  can be factorized into the  $B$  meson distribution amplitudes and the jet function, respectively, order by order. We complete the proof of the factorization theorem, which is graphically described in Fig. 2(c), and construct the jet function defined in Eq. (9).

#### IV. JET FUNCTION

We now perform threshold resummation of the double logarithms  $\alpha_s \ln^2 x$  in covariant gauge  $\partial \cdot A = 0$ , which have been collected into the jet function to all orders in the previous section. Threshold resummation for inclusive QCD processes has been studied intensively [9,10]. Here we shall adopt the framework developed in [12], which has been shown to lead to the same results as in [9,10]. First, we replace the vector  $n_+$  by  $n$ , which contains a plus component  $n^+ > 0$  and a (small) minus component. This replacement, regularizing the collinear pole, extracts the double logarithm as stated in Sec. II. The jet functions in terms of the dimensionless vectors  $n_+$  and  $n$  can be regarded as being defined in the different factorization schemes. The definition then involves three variable vectors: the Wilson line direction  $n$ , the large momentum  $P_2$ , and the spectator momentum  $k$ . We argue that the jet function depends on the Lorentz invariants,  $n \cdot k$ ,  $n \cdot P_2$  and  $P_2 \cdot k$ , since the other invariants  $P_2^2$  and  $k^2$  vanish, and  $n^2$  will be fixed below. The third invariant  $P_2 \cdot k$  is not independent, because it can be rewritten as  $n \cdot P_2 n \cdot k$  with  $n^2$  being a constant. We further argue that the scale invariance in  $n$ , as indicated by the Feynman rule associated with the eikonal line along  $n$ , implies that the jet function must depend on  $k$  through the ratio  $n \cdot k / n \cdot P_2$ .

The next step is to derive the evolution of the jet function in  $x$ , *i.e.*, in  $k^+ = x P_1^+$  by considering the derivative,

$$k^+ \frac{dJ}{dk^+} = \frac{n \cdot k}{P_2 \cdot k} P_2^\alpha \frac{dJ}{dn^\alpha}, \quad (19)$$

where we have applied the chain rule to relate the derivatives with respect to  $k$  and to  $n$ . The differentiation  $d/dn^\alpha$  operates on the eikonal line along  $n$ , giving

$$\frac{n \cdot k}{P_2 \cdot k} P_2^\alpha \frac{d}{dn^\alpha} \frac{n_\mu}{n \cdot l} = \frac{\hat{n}_\mu}{n \cdot l}, \quad (20)$$

with the special vertex,

$$\hat{n}_\mu = -\frac{n \cdot k}{P_2 \cdot k} \frac{P_2 \cdot l}{n \cdot l} n_\mu. \quad (21)$$

Another term in Eq. (21), proportional to  $P_2$ , is negligible, as the special vertex attaches inside of the jet function, which is dominated by momenta parallel to  $P_2$ .

The loop momentum  $l$  flowing through the special vertex does not generate a collinear divergence due to vanishing of the numerator  $P_2 \cdot l$  in this region. It is easy to confirm that the ultraviolet region of  $l$  does not produce  $\ln x$

either. Therefore, we concentrate on the factorization of the soft gluon emitted from the special vertex, which can be achieved by applying the eikonal approximation to internal quark propagators, leading to  $n_{-\nu}/n_- \cdot l$ . Following the reasoning in [12], the derivative of the jet function is written as

$$x \frac{dJ(x)}{dx} = -ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{n}_\mu}{n \cdot l} \frac{g^{\mu\nu}}{l^2} \frac{n_{-\nu}}{n_- \cdot l} J(x - l^+/P_1^+) , \quad (22)$$

where the argument of  $J$  in the integral arises from the invariant mass of the internal quark,  $(P_2 - k + l)^2 \approx -2(x - l^+/P_1^+)P_1 \cdot P_2$ . Performing the integration over  $l^-$  and  $l_T$ , we derive the evolution equation,

$$x \frac{dJ(x)}{dx} = \frac{\alpha_s}{2\pi} C_F \int_x^1 \frac{d\xi}{\xi - x} J(\xi) , \quad (23)$$

with the integration variable  $\xi$  corresponding to the plus component  $l^+$ .

Employing the variable change  $\xi = x/z$ , Eq. (23) becomes

$$x \frac{dJ(x)}{dx} = J^{(0)}(x) + \frac{\alpha_s}{2\pi} C_F \int_x^1 \frac{dz}{z} J(x/z) , \quad (24)$$

where the nonperturbative inhomogeneous term  $J^{(0)}(x)$  absorbs the soft pole from  $\xi \rightarrow x$  ( $z \rightarrow 1$ ). After removing this pole, the integral in the second term is infrared finite. It is interesting to observe that Eq. (24) is similar to the evolution equation for unintegrated parton distribution functions involved in inclusive QCD processes [20], which resums the same double logarithm  $\alpha_s \ln^2 x$ .

To solve Eq. (23), we perform the Mellin transformation from the momentum fraction ( $x$ ) space to the moment ( $N$ ) space:

$$\tilde{J}(N) \equiv \int_0^1 dx (1-x)^{N-1} J(x) . \quad (25)$$

The small  $x$  region then corresponds to the large  $N$  region. The left-hand side of Eq. (23) leads to

$$\int_0^1 dx (1-x)^{N-1} x \frac{dJ(x)}{dx} = -N \tilde{J}(N) + (N-1) \tilde{J}(N-1) \approx -\frac{d}{dN} [N \tilde{J}(N)] , \quad (26)$$

which is reasonable under the approximation  $dN \approx \Delta N = 1$ . The right-hand side is given by

$$\frac{\alpha_s}{2\pi} C_F \int_0^1 d\xi (1-\xi)^{N-1} J(\xi) \int_0^1 \frac{dw}{1-w} \left( \frac{1-\xi w}{1-\xi} \right)^{N-1} \approx \frac{\alpha_s}{2\pi} C_F \ln N \tilde{J}(N) , \quad (27)$$

where we have exchanged the sequence of the integrations over  $x$  and over  $\xi$ , made the variable change  $x = \xi w$ , and kept only the  $N$ -dependent term. The trick for extracting the  $N$  dependence from the integral over  $w$  in Eq. (27) is to consider its derivative with respect to  $N$ , and to insert the large  $N$  approximation for the  $\delta$ -function,  $\lim_{N \rightarrow \infty} N(1-y)^{N-1} = \delta(y)$  with  $y = \xi(1-w)/(1-\xi w)$ .

Equating Eqs. (26) and (27), we have the evolution equation in the moment space,

$$N \frac{d\tilde{J}(N)}{dN} = -\tilde{J}(N) - \frac{\alpha_s}{2\pi} C_F \ln N \tilde{J}(N) , \quad (28)$$

whose solution is the Sudakov factor,

$$\tilde{J}(N) = \frac{1}{N} \exp \left( -\frac{1}{4} \gamma_K \ln^2 N \right) , \quad (29)$$

with the anomalous dimension  $\gamma_K = \alpha_s C_F / \pi$ . The factor  $1/N$  can be regarded as the Mellin transformation of the initial condition  $J^{(0)} = 1$ . Comparing Eq. (29) with the resummation for an inclusive jet [9,10], the resummation for an exclusive jet is suppressed by a factor  $1/N$ , and the corresponding anomalous dimension is down by a factor 2.

By means of the inverse Mellin transformation, the jet function in the momentum fraction space is written as

$$J(x) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (1-x)^{-N} \tilde{J}(N) , \quad (30)$$

with  $c$  being an arbitrary real constant larger than all the real parts of the poles of the integrand. Since  $\tilde{J}(N)$  contains a branch cut along the negative real axis on the complex  $N$  plane, Eq. (30) reduces to

$$J(x) = -\exp\left(\frac{\pi}{4}\alpha_s C_F\right) \int_{-\infty}^{\infty} \frac{dt}{\pi} (1-x)^{\exp(t)} \sin\left(\frac{1}{2}\alpha_s C_F t\right) \exp\left(-\frac{\alpha_s}{4\pi} C_F t^2\right), \quad (31)$$

where the variable change  $N = \exp(t + i\pi)$  ( $N = \exp(t - i\pi)$ ) has been adopted for the piece of contour above (below) the branch cut as shown in Fig. 3. Note that the above expression holds only for  $\alpha_s > 0$ . For  $\alpha_s = 0$ , the residue associated with the  $N = 0$  pole should be included. It is trivial to check that  $J(x)$  is normalized to unity,  $\int J(x) dx = \tilde{J}(1) = 1$  [21].

Obviously, Eq. (31) vanishes at  $x \rightarrow 0$ , because the integrand is an odd function in  $t$ , and at  $x \rightarrow 1$  due to the factor  $(1-x)^{\exp(t)}$ . The latter property is the consequence of the extrapolation of the Sudakov factor to the low  $N$  region. Moreover, Eq. (31) provides suppression near the end point  $x \rightarrow 0$ , which is stronger than any power of  $x$ . This is understood from vanishing of all the derivatives of Eq. (31) with respect to  $x$  at  $x \rightarrow 0$ . For example, the first derivative gives

$$\frac{d}{dx} J(x)|_{x \rightarrow 0} = -\exp\left(\frac{\pi}{4}\alpha_s C_F + \frac{\pi}{\alpha_s C_F}\right) \int_{-\infty}^{\infty} \frac{dt}{\pi} e^t \sin\left(\frac{1}{2}\alpha_s C_F t\right) \exp\left(-\frac{\alpha_s}{4\pi} C_F t^2\right), \quad (32)$$

where the variable change  $t \rightarrow t + 2\pi/(\alpha_s C_F)$  has been made. The integrand in Eq. (32) is also an odd function in  $t$ , and the integral diminishes. To the accuracy of the next-to-leading logarithms, the running of the coupling constant  $\alpha_s$  should be taken into account, and Eq. (31) will be modified. However, the above features remain.

## V. SEMILEPTONIC DECAY

In this section we extend the above formalism to the semileptonic decay  $B \rightarrow \pi l \bar{\nu}$  in the fast recoil region of the pion. The  $B$  meson momentum  $P_1$  is the same as in the decay  $B \rightarrow \gamma l \bar{\nu}$ , and the pion momentum  $P_2$  is the same as the photon momentum. Leading-twist factorization theorem for the  $B \rightarrow \pi$  form factor  $f_+(q^2)$  in the standard definition has been proved in [7],

$$f_+(q^2) = \sum_{m=+,-} \phi_m(x_1) \otimes H_m(x_1, x_2, \eta) \otimes \phi_\pi(x_2), \quad (33)$$

which holds in the region with  $x_1 \sim O(\bar{\Lambda}/M_B)$  and with  $x_2 \sim O(1)$ .

Since Fig. 4(a), proportional to  $1/(x_1 x_2^2)$ , is more singular at small  $x_2$ , we consider the end-point region with  $x_2 \sim O(\bar{\Lambda}/M_B)$ , where the internal  $b$  quark propagator scales like  $1/(\bar{\Lambda} M_B)$ . Part of  $O(\alpha_s)$  corrections to Fig. 4(a) are shown in Figs. 4(c)-(e), among which Fig. 4(c) generates the double logarithm  $\alpha_s \ln^2 x_2$  from the collinear region with the loop momentum parallel to  $P_2$ . Figures 4(d) and 4(e) involve only the single logarithm. We emphasize that the double logarithm discussed here for Fig. 4(a) differs from that in [4], which concerns  $\alpha_s \ln^2 x_1$ . As stated above, Fig. 4(a) is less singular at  $x_1 \rightarrow 0$ , and  $\alpha_s \ln^2 x_1$  is not relevant. The double logarithm in Fig. 4(c) is simply extracted by eikonalizing the light quark line, which flows into the pion, and the  $b$  quark line according to Eq. (3):

$$\begin{aligned} J_{\parallel}^{(1)} &= -ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{n_+ \cdot n_-}{n_+ \cdot (l - x_2 P_2) n_- \cdot l l^2}, \\ &= -\frac{\alpha_s}{4\pi} C_F \ln^2 x_2 + \dots \end{aligned} \quad (34)$$

Similarly, the collinear pole has been regularized by including a small plus component for  $n_-$ . It is observed that the double logarithm in Eq. (34) is the same as in Eq. (6).

The all-order factorization of the jet function from the decay  $B \rightarrow \gamma l \bar{\nu}$  can be proved following the procedures in Sec. III. It is also easy to show that this jet function obeys the evolution equation in Eq. (23) by identifying the correspondence of  $x_2 P_2$  and  $n_+$  to  $k$  and  $P_2$  in Sec. IV, respectively. Hence, the threshold resummation leads to a result the same as Eq. (31). That is, the Sudakov factor is universal.

The analysis for Fig. 4(b) is similar to that for the decay  $B \rightarrow \gamma l \bar{\nu}$ . In the end-point region with  $x_1 \sim O(\bar{\Lambda}^2/M_B^2)$ , additional collinear divergences associated with the internal light quark are produced in Figs. 4(f)-4(h). Figure 4(f) gives the double logarithm  $\alpha_s \ln^2 x_1$ , whose factorization is the same as of Fig. 1(c). Note that Fig. 5 has been considered as the dominant source of the double logarithm  $\alpha_s \ln^2 x_1$  associated with Fig. 4(b) [4]. It will be

demonstrated explicitly in the Appendix A that Fig. 5 in fact contains only the single logarithm  $\alpha_s \ln x_1$ . The result in [4] might be attributed to an inappropriate collinear approximation. Therefore, the universality of the double-logarithm resummation indeed holds.

At last, we investigate the resummation effect on the  $B \rightarrow \pi$  form factor  $f_+(q^2)$ , whose factorization formula is given by

$$f_+(q^2) = \frac{\pi\alpha_s C_F f_B f_\pi}{\eta^2 M_B^2 N_c} \int dx_1 dx_2 \phi_B(x_1) \left[ \frac{J(x_2)}{x_1 x_2^2} (1 + x_2 \eta) - \frac{J(x_1)}{x_1 x_2} (1 - \eta) \right] \phi_\pi(x_2), \quad (35)$$

with  $N_c = 3$  being the number of colors. We employ the models [22,23],

$$\begin{aligned} \phi_\pi(x) &= 6x(1-x)\{1 + 0.66[5(1-2x)^2 - 1]\}, \\ \phi_B(x) &\equiv \frac{1}{2}[\phi_+(x) - \phi_-(x)] = N_B \sqrt{x(1-x)} \exp \left[ -\frac{1}{2} \left( \frac{x M_B}{\omega_B} \right)^2 \right], \end{aligned} \quad (36)$$

with the shape parameter  $\omega_B$  and the normalization constant  $N_B$  determined by  $\int \phi_B(x) dx = 1$ . Another  $B$  meson distribution amplitude  $\bar{\phi}_B(x) = [\phi_+(x) - \phi_-(x)]/2$  is power-suppressed, since its first nonvanishing moment starts with  $O(1/M_B)$  [16]. It has been confirmed, contrary to the conclusion in [24], that the contribution from  $\bar{\phi}_B$  is less important than that from  $\phi_B$  [25]. For the purpose of this work, it is sufficient to work on a single  $B$  meson distribution amplitude. If  $J(x)$  is excluded, the first term in Eq. (35) is logarithmically divergent. With the threshold resummation, the form factor is calculable without introducing any infrared cutoffs [3–5].

Choosing  $\alpha_s = 0.4$ ,  $M_B = 5.28$  GeV, and the decay constants  $f_B = 190$  MeV and  $f_\pi = 130$  MeV, we obtain  $f_+(q^2)$  for  $\omega_B = 0.3(0.4)$  GeV,

$$f_+(0) = 0.15(0.12), \quad f_+(2 \text{ GeV}^2) = 0.18(0.13), \quad f_+(4 \text{ GeV}^2) = 0.21(0.15). \quad (37)$$

The variation of  $f_+$  for  $\alpha_s = 0.3$ - $0.5$  is less than 10%, because the change of  $J$  is compensated by that of  $\alpha_s$  in the overall coefficient in Eq. (35). The difference between the above values and the expected one  $f_+(0) \sim 0.3$  can be resolved by taking into account higher-twist contributions [16]. Including two-parton twist-3 pion distribution amplitudes, which are finite at the end point [22], the singularities in the  $B \rightarrow \pi$  form factor become linear. In this case threshold resummation is even more crucial.

## VI. CONCLUSION

In this paper we have shown that the double logarithmic corrections  $\alpha_s \ln^2 x$  appear in exclusive  $B$  meson decays. When the end-point region with a momentum fraction  $x \rightarrow 0$  is important, these double logarithms need to be organized to all orders in order to justify perturbative expansion. The double logarithms, associated with internal particles which almost go on mass shell, are of the collinear nature. The PQCD factorization theorem for exclusive  $B$  meson decays at the end point then demands the introduction of a quark jet function into decay amplitudes. The factorization of the jet function from the  $B \rightarrow \gamma l \bar{\nu}$  mode has been proved rigorously. The proof for the factorization from other modes, such as the semileptonic decay  $B \rightarrow \pi l \bar{\nu}$ , is similar. It has been shown that the jet function is defined as a matrix element of a quark field attached by a Wilson line, based on which threshold resummation can be performed. It is interesting to achieve the above proof in the framework of the soft-collinear effective theory [26].

We have derived the evolution equation for the jet function, which is solvable in the large  $N$  limit in the Mellin space. The solution is a universal, *i.e.*, process-independent, Sudakov factor. The qualitative behavior of this Sudakov factor has analyzed and found to decrease quickly at  $x \rightarrow 0$ . It has been demonstrated that Sudakov suppression is strong enough to smear the end-point singularity in the  $B \rightarrow \pi$  form factor and leads to reasonable predictions. We conclude that in a self-consistent PQCD analysis of the heavy-to-light transition form factors, the end-point singularities do not exist. In a future work we shall extend the formalism developed here to more complicated nonleptonic  $B$  meson decays. Threshold resummation for various topologies of decay amplitudes, such as annihilation and nonfactorizable ones, will be studied.

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## APPENDIX A: TRIPLE-GLUON CORRECTION

In this Appendix we show that Fig. 5 produces only the single logarithm  $\alpha_s \ln x_1$ . The loop integral is written as

$$I = g^4 \int \frac{d^4 l}{(2\pi)^4} \text{tr} \left[ \gamma_\delta \frac{\not{P}_2}{4N_c} \gamma_\beta \frac{\not{P}_2 - \not{k}_1 + \not{l}}{(P_2 - k_1 + l)^2} \gamma^\mu \right. \\ \times \frac{\not{P}_1 - \not{k}_1 + \not{l} + M_B}{(P_1 - k_1 + l)^2 - M_B^2} \gamma_\alpha \frac{(\not{P}_1 + M_B) \gamma_5}{4N_c} \left. \right] \\ \times \frac{\Gamma^{\alpha\beta\delta} f^{abc} \text{tr}(T^a T^b T^c)}{(k_1 - x_2 P_2 - l)^2 l^2 (x_2 P_2 - k_1)^2}, \quad (\text{A1})$$

with the triple-gluon vertex,

$$\Gamma^{\alpha\beta\delta} = (2x_2 P_2 - 2k_1 + l)^\alpha g^{\beta\delta} + (k_1 - x_2 P_2 - 2l)^\delta g^{\alpha\beta} + (l - x_2 P_2 + k_1)^\beta g^{\alpha\delta}, \quad (\text{A2})$$

the anti-symmetric tensor  $f^{abc}$ , and the color matrices  $T^{a,b,c}$ .

We concentrate on the first term in Eq. (A2), since the calculation of the second and third terms is similar. If the loop momentum  $l$  in the quark propagator  $1/(P_2 - k_1 + l)^2$  is neglected, Eq. (A1) gives the double logarithm  $\alpha_s \ln^2 x_1$  as obtained in [4]. However, this approximation is not appropriate for the collection of the collinear divergence, for which  $l$  is of the same order as  $P_2$ . Keeping the  $l$  dependence, and using the identity  $f^{abc} \text{tr}(T^a T^b T^c) = 6i$ , we have

$$I_1 = -\frac{ig^4}{4N_c} \frac{\text{tr}(\not{P}_2 \not{k}_1 \gamma^\mu \not{P}_1)}{(x_2 P_2 - k_1)^2} \\ \times \int \frac{d^4 l}{(2\pi)^4} \frac{v \cdot (2x_2 P_2 - 2k_1 + l)}{v \cdot l (P_2 - k_1 + l)^2 (k_1 - x_2 P_2 - l)^2 l^2}. \quad (\text{A3})$$

The contribution proportional to  $M_B^2$  in Eq. (A1) is suppressed by a power of  $x_1$  compared to Eq. (A3). For a similar reason, the term  $v \cdot k_1$  in Eq. (A3) is also negligible. The term  $v \cdot l$  in the numerator does not lead to a collinear divergence, because the poles of the corresponding denominator  $(P_2 - k_1 + l)^2 (k_1 - x_2 P_2 - l)^2 l^2$  in the  $l^-$  plane are not of the pinched type.

Doing the contour integral with the residue  $l^- = -l^+ - i\epsilon$  (for  $l^+ < 0$ ), Eq. (A3) becomes

$$I_1 = H_{4b}^{(0)\mu} \frac{4g^2}{(2\pi)^3 C_F} \int_{-\infty}^0 dl^+ \int_{-\infty}^{\infty} d^2 l_T \\ \times \frac{x_2 P_2^- (P_2 - k_1)^2}{(l_T^2 + 2l^{+2} - 2P_2^- l^+ + 2P_2^- k_1^+)(l_T^2 + 2l^{+2} - 2x_2 P_2^- l^+ + 2x_2 P_2^- k_1^+)(l_T^2 + 2l^{+2})}, \quad (\text{A4})$$

where

$$H_{4b}^{(0)\mu} = \frac{g^2 C_F \text{tr}(\not{P}_2 \not{k}_1 \gamma^\mu \not{P}_1)}{8N_c (x_2 P_2 - k_1)^2 (P_2 - k_1)^2}, \quad (\text{A5})$$

is the lowest-order amplitude from Fig. 4(b). The integration over  $l_T$  leads to

$$I_1 = H_{4b}^{(0)\mu} \frac{\alpha_s}{4\pi} C_A x_2 k_1^+ \int_0^\infty dl^+ \left[ \frac{\ln(l^{+2}/M_B^2 + \eta l^+/M_B + x_1 \eta)}{(1 - x_2)(l^+ + k_1^+)^2} \right. \\ \left. - \frac{\ln(l^{+2}/M_B^2 + x_2 \eta l^+/M_B + x_1 x_2 \eta)}{x_2(1 - x_2)(l^+ + k_1^+)^2} + \frac{\ln(l^{+2}/M_B^2)}{x_2(l^+ + k_1^+)^2} \right], \quad (\text{A6})$$

with the color factor  $C_A = 3$ .

The terms  $l^{+2}/M_B^2$  in the first two logarithms, giving contributions suppressed by a power of  $x_1$  compared to those from  $l^+/M_B$ , can be dropped. Each term in Eq. (A6) then produces only the single logarithms  $\alpha_s \ln x_1$ :

$$I_1 = H_{4b}^{(0)\mu} \frac{\alpha_s}{4\pi} C_A x_2 \left[ \frac{\ln x_1}{1 - x_2} - \frac{\ln x_1}{x_2(1 - x_2)} + 2 \frac{\ln x_1}{x_2} \right], \\ = H_{4b}^{(0)\mu} \frac{\alpha_s}{4\pi} C_A \ln x_1. \quad (\text{A7})$$

In the above expression the constant terms independent of  $\ln x_1$  are not displayed. It can be shown in a similar way that the other two terms in Eq. (A2) involve only the single logarithms.

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## Figure Captions

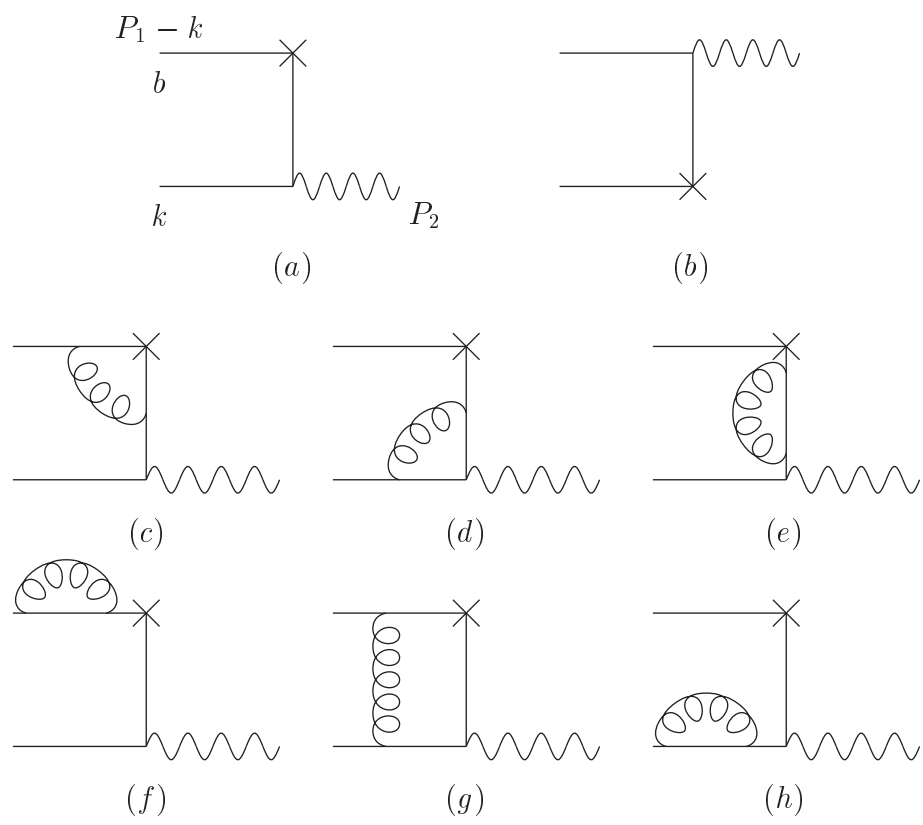
**Fig. 1.** (a) and (b) Lowest-order diagrams for the decay  $B \rightarrow \gamma l \bar{\nu}$ . (c)-(h)  $O(\alpha_s)$  corrections to Fig. 1(a).

**Fig. 2.** (a) The Ward identity. (b) Factorization of the  $O(\alpha_s)$  diagrams as a consequence of (a). (c) Factorization of the  $B \rightarrow \gamma l \bar{\nu}$  decay amplitude in the end-point region.

**Fig. 3.** Contour for the inverse Mellin transformation of the jet function.

**Fig. 4.** (a) and (b) Lowest-order diagrams for the decay  $B \rightarrow \pi l \bar{\nu}$ . (c)-(e) [(f)-(h)] Part of  $O(\alpha_s)$  corrections to Fig. 4(a) [Fig. 4(b)].

**Fig. 5.** Triple-gluon correction to Fig. 4(b).



**FIG. 1**

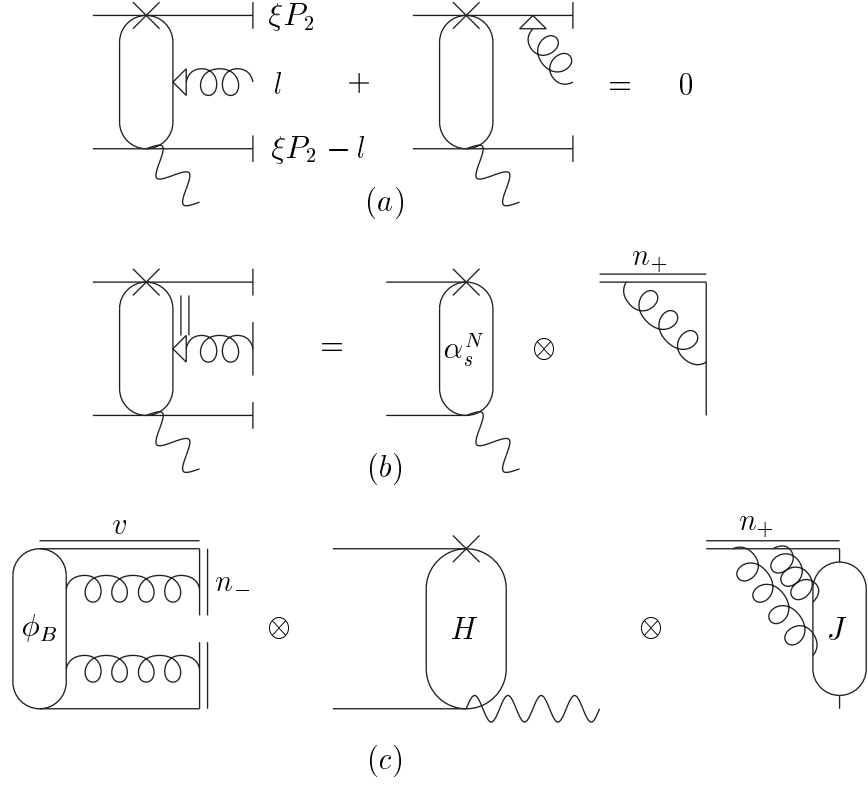
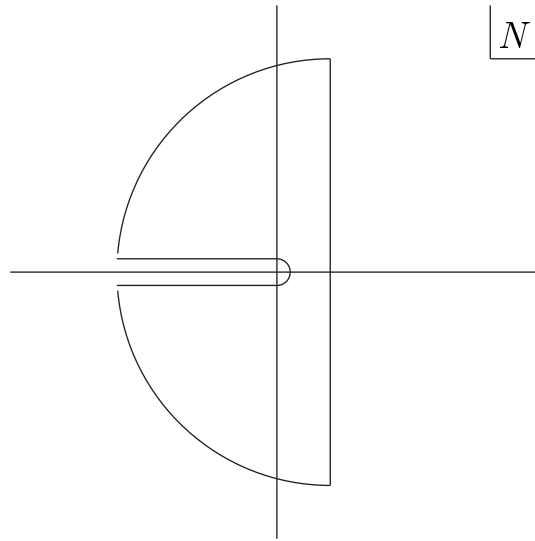
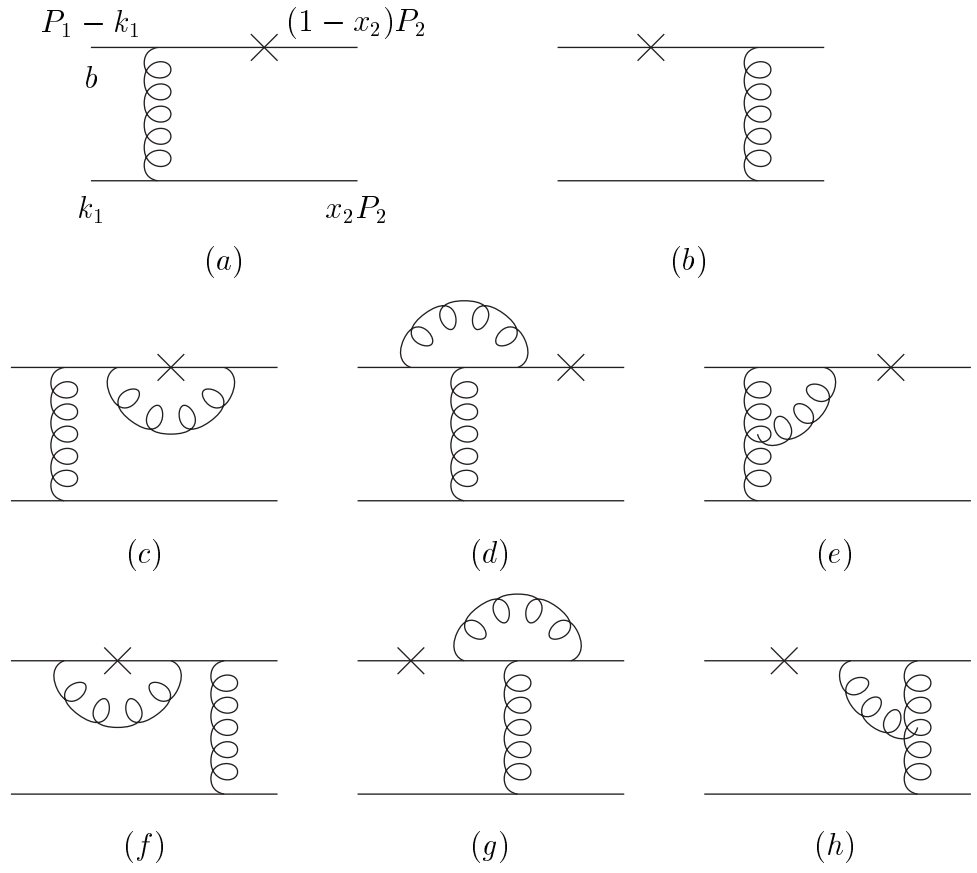


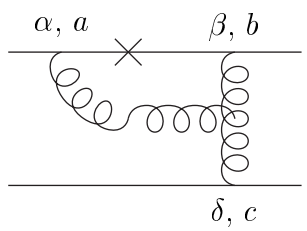
FIG. 2



**FIG. 3**



**FIG. 4**



**FIG. 5**